

## ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 4

DEADLINE: FRIDAY, NOVEMBER 10TH

**Problem 1.** Use a Serre spectral sequence to compute  $\pi_5(S^3, *)$ .

Hint 1: Use the Whitehead tower for  $S^3$ .

Hint 2: There are different ways to proceed, but you might need to know  $H_n(K(\mathbb{Z}/2, 3); \mathbb{Z})$ , for  $n \leq 5$ . For this, recall that

$$H_n(K(\mathbb{Z}/2, 1); \mathbb{Z}) \cong \begin{cases} \mathbb{Z} & \text{if } n = 0 \\ \mathbb{Z}/2, & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even and } n > 0 \end{cases}$$

Start by using the path-loop fibration of  $K(\mathbb{Z}/2, 2)$  to compute  $H_n(K(\mathbb{Z}/2, 2); \mathbb{Z})$  in low degrees. Then repeat the process for the path-loop fibration of  $K(\mathbb{Z}/2, 3)$ .

**Problem 2.** Compute the cohomology of the space  $\text{map}(S^1, S^3)$  of continuous (not necessarily basepoint-preserving) maps  $f : S^1 \rightarrow S^3$ .